COMPARISON OF IMAGE ENHANCEMENT TECHNIQUES ON SQUARE AND HEXAGONAL LATTICE STRUCTURES

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ABSTRACT: Enhancement of digital images plays a vital role in the processing of digital imaging. Especially data obtained from satellite, remote sensing, medical imaging, foreignsic science etc. Literature revealed that a number of enhancement techniques are available for square lattices, but very less work is available for the enhancement of hexagonal lattice structure. In this paper our research work is implementation of existing enhancement techniques on another structure called hexagonal lattice structure. From the simulation results, it has been investigated that the enhancement of hexagonal pixels shows high Peak Signal to Noise Ratio (PSNR) and Low Mean Square Error (MSE) and better image quality as compared to square lattice structure.

KEYWORDS: Image Enhancement, Hexagonal Image Processing, Image Enhancement Techniques.

INTRODUCTION

Hexagonal lattices have been of interest to humans in the field of image processing from many decades. The top most advantage of the hexagonal representation is its resemblance with Human Vision System. Its computational power in the field of image processing pushes the hexagonal structure for intelligent vision. Dozens of researches have described the advantages of hexagonal structure [6, 10, 13]. Also the lattice geometry of hexagonal lattice has some advantages over the square lattice like a higher degree of circular symmetry, uniform fittings of pixels, reduction in complex calculations while processing, enhancing image quality, consistent connectivity, additional equidistant neighbours, isoperimetric, uniform connectivity, reflection symmetry. So images with the square pixels are lesser pertinent to the vision process. In spite of these advantages, there is no such direct hardware that can capture hexagonal images directly. Conventional acquisition devices like camera, acquire images only on square pixels, but our approach is to manipulate square sampled images via software to produce hexagonal sample images. The conversion of square pixel to hexagonal pixel structure can be implemented using a number of techniques [7, 9, 11].

The paper is organized as follows. Section II deals with existing image enhancement techniques. In section III conversion of hexagonal lattice structure from square lattice is implemented using spiral addressing scheme (SAS), followed by enhancement. Section IV evaluates the performance of image quality in terms of PSNR, MSE & BER. Finally experimental results and discussion are provided in section V.

IMAGE ENHANCEMENT TECHNIQUES

Image enhancement is a process by which the minute information of an object can be obtained. Enhancement of digital images in digital image processing is an important step; as detailed information can be obtained from the same image. But image enhancement is subjective. The scene captured by an eye and clicked by different cameras and at different settings may vary. This is because of different configurations of cameras, different lighting conditions at different angles. There are a number of techniques available that can enhance the image to make it identical with the human visual system. Some important image enhancement techniques are discussed in the following section.

Single Scale Retinex Algorithm

Retinex improves visual quality of an image when lighting conditions are not good. Human eyes can see a color correctly when light is low, but all cameras and video cams can't manage this well during varying lighting conditions. Retinex stands for Retina + cortex. Retinex is a method which is used for bridging the gap between images and the human observation of scenes [4].



Figure 1: Block diagram of Single Scale Retinex [16]

Fig. 1 shows the block diagram of single scale retinex technique for enhancement of digital images. The single scale retinex algorithm can be implemented using the following equation:

$$SSR(x, y) = \log[I(x, y)] - \log[T]$$
(1)

$$T = [G_n(x, y) * I(x, y)]$$
(2)

Where I(x, y): Input image; SSR(x, y): Retinex output; $G_n(x, y)$ = Gaussian surround function; ' * ' indicates convolution operator. The surround function is a low pass filter which is used to remove the noise component from an image and is given by the equation:

$$G_n(x, y) = C_n * e^{-x^2 + y^2/2\sigma^2}$$
(3)

Where

$$C_n = \frac{1}{\sum_{i=1}^M \sum_{j=1}^N e^{-x^2 + y^2/2\sigma^2}}$$
(4)

x, *y* are the spatial coordinates, M, N represents size of image and σ is the Gaussian surround space constant, and it is referred to as the scale of the SSR. C_n is selected such that

$$\iint F(x, y) dx dy = 1 \tag{5}$$

Multi Scale Retinex Algorithm

Multi Scale Retinex is a generalization of Single Scale Retinex. It can be obtained by weighted sum of several different Single Scale Retinex methods. Fig. 2 shows the block diagram of Multi

Scale Retinex algorithm; that has been obtained with the help of single scale retinex and is given by the equation:

$$MSR(x, y) = \sum_{n=1}^{N} W_n SSR(x, y)$$
(6)

Where MSR(x, y) is MSR output; W_n is a weighting factor which has value between 0 and 1; sum of all the weights should be 1. Here the value of $W_n = 1/3$. N indicates number of scale, SSR(x, y) is output of Single Scale Retinex method and can be represented by the following Equation:

$$SSR(x, y) = \log[I(x, y)] - \log[G_n(x, y) * I(x, y)]$$
(7)



Figure 2: Block diagram of Multi Scale Retinex

Multi-Scale Retinex with Color Restoration (MSRCR)

The Multi Scale Retinex image contains features from all three scales simultaneously; in addition with dynamic range and tonal rendition. But in practice MSR is weak in tonal rendition and does not provide good tonal rendition [8]. A method to deal with this problem is Multi-Scale Retinex with Color Restoration. The color restoration process is used to provide good color rendition for any degree of graying and preserves a reasonable degree of color constancy and it can be implemented using the following equations:

$$R_{MSRCRi}(x, y) = C_i(x, y)R_{MSRi}(x, y)$$
(8)

Where $C_i(x, y)$ is given as:

$$C_i(x,y) = f[I'_i(x,y)] \tag{9}$$

and
$$I_i(x, y) = I_i(x, y)/\sum_{i=1}^{i} I_i(x, y)$$
 (10)
 $C_i(x, y) = \beta \log[\alpha I_i'(x, y)]$ (11)

$$\mathcal{L}_i(x, y) = \beta \log[\alpha I_i(x, y)]$$

Where α controls the strength of nonlinearity and β is the gain constant. The best results are obtained with value of α and β i.e. 125 and 2.65 respectively.

Homomorphic Filtering

Images are sometimes been acquired under poor illumination. Under this condition, some uniform regions will appear brighter on some areas and darker on others. This undesired situation will leads to several severe problem in computer vision based system. The pixels might be misclassified, leading to wrong segmentation results, and therefore contribute to inaccurate evaluation or analysis from the system. Therefore, it is very crucial to process this type of images first before they are fed into the system. One of the popular methods used to enhance or restore the degraded images by uneven illumination is by using homomorphic filtering. This technique uses illumination-reflectance model in its operation. Homomorphic filtering is a generalized technique for image enhancement. It simultaneously normalizes the brightness across an image and increases contrast. An image can be expressed as the product of illumination and reflectance:

$$F(x,y) = I(x,y).R(x,y)$$
(12)

Taking Natural log of equation (12) gives

$$G = \ln(F(x, y)) = \ln(I(x, y)) + \ln(R(x, y))$$
(13)

Calculate Fourier Transform of equation (13) $\Im\{G(x, y)\} = \Im\{\ln(I(x, y))\} + \Im\{\ln(R(x, y))\}$

Equation (14) can be represented as:

$$G(u, v) = I_l(u, v) + R_l(u, v)$$
(15)

$$S(u, v) = H(u, v) \cdot G(u, v) = H(u, v) [I_l(u, v) + R_l(u, v)]$$
(16)

(14)

(20)

Where

$$H(u, v) = (\gamma_H - \gamma_L)[1 - \exp\left[-c\left(\frac{D^2(u,v)}{D_0^2}\right)\right] + \gamma_L]$$

$$(17)$$

$$S(x, y) = S^{-1}\{H(u, v)\}$$
(18)
$$S(x, y) = \Im^{-1}\{H(u, v)[I_1(u, v) + R_1(u, v)]\}$$
(19)

$$C'(x,y) + R'(x,y)$$

$$S'(x,y) = \exp(S(x,y)) = \exp(C'(x,y)) \cdot \exp(R'(x,y))$$
(21)

$$S'(x, y) = C''(x, y) \cdot R''(x, y)$$
(22)

C''(x, y) & R''(x, y) are the illumination and reflectance of the enhanced image. The illumination component tends to vary slowly across the image and the reflectance tends to vary rapidly; particularly at junctions of dissimilar objects. Fig. 3 shows the block diagram of homomorphic filtering.



Figure 3: Block diagram of homomorphic filtering

Gabor filter

Frequency and orientation representations of Gabor filters are similar to those of the human visual system, and they have been found to be particularly appropriate for texture representation and discrimination [15]. In the spatial domain, a 2D Gabor filter is a Gaussian kernel function modulated by a sinusoidal plane wave. It can be written as:

h(x,y) = s(x,y).g(x,y)

(23)

(24)

s(x,y) is a complex sinusoid known as carrier; g(x,y) is a 2-D Gaussian shape function known as envelope. The complex sinusoid is defined as follows:

 $s(x, y) = e^{-j2\pi(u_0 x + v_0 y)}$

The 2-D Gaussian function is defined as follows:

$$g(x, y) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2} \left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)}$$
(25)

The 2D gabor filter can be written as

$$h(x, y) = e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)} \cdot e^{-j2\pi(u_0 x + v_0 y)}$$
(26)

SIMULATION AND ENHANCEMENT OF HEXAGONAL PIXEL

Simulation Work

From past so many years, the work is in process for simulation of hexagonal grid from rectangular grid structure. The simulation process techniques include pseudo hexagonal pixel, mimic hexagonal pixel, virtual hexagonal pixel, and spiral addressing structure. The use of these techniques on rectangular grid provides a platform, where a research on hexagonal pixel structure is possible using existing graphics and computer vision. In this paper, our motive is to simulate hexagonal grid from rectangular pixel structure using spiral addressing scheme, due to its various advantages over other existing schemes; and final task is the implementation of image enhancement techniques on hexagonal grid structure. Flow chart of the proposed work is as follows:



Figure 4: Flow Chart of proposed work

Simulation of Hexagonal Pixels

To construct hexagonal pixels, each square pixel is first separated in 7*7 smaller pixels called subpixels. Hexagonal pixels are designed using these 56 Sub-pixels and the structure is given in the following fig. 5.

| | | х | х | х | х | х | | |
|---|---|---|---|---|---|---|---|---|
| | x | х | х | х | х | x | x | |
| | х | х | х | х | х | х | x | |
| х | х | x | х | х | X | x | х | х |
| х | х | х | х | х | x | x | x | x |
| | x | х | х | х | х | х | x | |
| | х | х | х | х | x | х | x | |
| | | x | х | х | x | x | | |

Figure 5. The structure of a single hexagonal pixel

The design shown in fig. 5 implies the structure of one individual hexagonal pixel. A cluster of seven such hexagonal pixels can be found in the following fig. 6(a). The addressing of these hexagonal pixels is done like spiral architecture. The address of central pixel is 0 and its neighbouring 6 pixels have address 1, 2, 3, 4, 5, 6. The complete addressing of hexagonal pixels is done by using base seven addresses only. This means that after 6 the next address will be 10. Fig. 6(b) shows the complete addressing of cluster of $7^2 = 49$ square pixels. This hexagonal structure retains the property of equal distance as all the neighbouring pixels are equidistant from their central pixel. And this type of construction hardly introduces image distortion.



Figure 6(a) Cluster of seven hexagonal pixels [6]; (b) Spiral Architecture with spiral addressing for a cluster of 49 hexagonal pixels [17].

If D (x) is used to denote the location of the hexagonal pixel with spiral address architecture. Thus $D(0) = [0 \ 0]$. From Fig. 6(a), it is easy to see that $D(1) = [8 \ 0]$, $D(2) = [4 \ -7]$, $D(3) = [-4 \ -7]$, $D(4) = [-8 \ 0]$, $D(5) = [-4 \ 7]$ and $D(6) = [4 \ 7]$. The shift for addresses 0 to 6 are base cases for the recursive algorithm. The algorithm for multiples of 10 is given by:

$$D(\mathbf{x} \times 10^{i}) = D(\mathbf{x} \times 10^{i-1}) + 2D((\mathbf{x}+1) \times 10^{i-1})$$
(27)

$$D(6 \times 10^{i}) = D(6 \times 10^{i-1}) + 2D(10^{i-1})$$
(28)

For
$$i = 1, 2..., x = 1, 2..., 5.$$
 (29)

While the location of the pixels with a given spiral address

$$x_{n_i} x_{n-1_i} \dots a_l, (x_i = 0, 1, 2 \dots 6. \text{ For } i = 1, 2 \dots, n.)$$
 (30)

Can be obtained by

$$D(x_n x_{n-1}...a_l) = \sum_{i=1}^n D(x_i \times 10^{i-l})$$
(31)

Finally the image obtained after conversion is shown below as:



Figure 7 (a) Original square pixel image; Conversion to hexagonal pixel image (b) using 7 pixels (c) using 7^2 pixels (d) using 7^3 pixels (e) using 7^4 pixels (f) using 7^5 pixels

EVALUATION OF IMAGE QUALITY

The results are calculated on four different images i.e. (a) Foggy Road Image (b) Brain MRI Image (c) Flower Image (d) Face Image. Figure 8 (first row) and Figure 9 (first row) show all the original images in square pixel structure and their conversion to hexagonal pixel structure. The other rows show the enhancement of these square and hexagonal pixel structure using Single Scale Retinex techniques, Multi Scale Retinex, Multi Scale Retinex with Color Restoration, Homomorphic Filtering and Gabor Filtering.

| | Square Pixel Image | Hexagonal Pixel | Square Pixel | Hexagonal Pixel |
|-------------------|--------------------|-----------------|--------------|-----------------|
| | | Image | Image | Image |
| Original Image | | | | |
| | (a) | (b) | (c) | (d) |



Figure 8: Square pixel and its conversion to hexagonal pixel images of foggy road and brain MRI (top row); their enhancement using SSR, MSR, MSRCR, Homomorphic filtering, Gabor filter (consecutive rows)

| | Square Pixel Image | Hexagonal Pixel Image | Square Pixel Image | Hexagonal Pixel Image |
|--|-----------------------|--------------------------|-----------------------|--------------------------|
| Original Image | NO 1 | | · | 2 |
| 0:1 | (a) | (b) | (c) | (d) |
| scale Retinex | NOV | NO A | Q | Q |
| | (e) | (f) | (g) | (h) |
| e Retinex | - Color | North State | 2 | 2 |
| | (i) | (j) | (k) | (1) |
| Multiscal e Retinex with Color Restoratio n | Star. | Nov. | 2 | 2 |
| Hama | (m) | (n) | (0) | (p) |
| morphic Filtering | | Nº25 | | |
| | (q) | (r) | (s) | (t) |
| Gabor Filter | (u) | (v) | (w) | (x) |

Figure 9: Square pixel and its conversion to hexagonal pixel images of flower and face (top row); their enhancement using SSR, MSR, MSRCR, homomorphic filtering, Gabor filter (consecutive rows)

The quality of an image is evaluated using the Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE) and Bit Error Rate (BER) approach. The higher the PSNR value, the better quality of the image is obtained. PSNR (peak signal to noise ratio) of an image of size $m \times n$ is calculated using the equation as follows:

$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE}\right)$$
(32)

Where MSE is the mean square error and is calculated as

$$MSE = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \frac{(A_{ij} - B_{ij})^2}{m \times n}$$
(33)
and bit error rate is obtained as
$$BER = \frac{1}{PSNR}$$
(34)

Table 1, Table 2, Table 3, Table 4 outcome the values of MSR, BER, PSNR of four images i.e. Foggy Road Image, Brain MRI Image, Flower Image and Face Image respectively applied on square pixel enhanced structures and hexagonal pixel enhanced structures.

| Table 1: Performance | comparison | of image | enhancement | techniques | on | foggy | road | image | using |
|--------------------------|--------------|--------------|-------------|------------|----|-------|------|-------|-------|
| square pixel structure a | and hexagona | al pixel str | ructure | | | | | | |

| | Mean Square Error (MSE) | | Bit Error | Rate (BER) | Peak Signal to Noise Ratio (PSNR) | |
|---|-------------------------|--------------------------|-----------------|--------------------------|--------------------------------------|--------------------|
| Foggy Road Image Results | Square pixel image | hexagonal pixel image | Square pixel | hexagonal pixel image | Square pixel | hexagonal pixel |
| Single coole ratingy | 0.0051 | 0.0047 | 0.0141 | 0.0140 | 71.0660 | 1111age |
| Single scale letillex | 0.0051 | 0.0047 | 0.0141 | 0.0140 | /1.0009 | /1.4209 |
| Multiscale retinex | 0.0025 | 0.0021 | 0.0135 | 0.0133 | 74.1742 | 74.9879 |
| Multiscale retinex with color restoration | 0.0084 | 0.0076 | 0.0145 | 0.0144 | 68.8960 | 69.3507 |
| Homomorphic filtering | 0.1266 | 0.1327 | 0.0175 | 0.0176 | 57.1064 | 56.9030 |
| Gabor filter | 7.69×10^{-4} | 3.87×10 ⁻⁴ | 0.0126 | <u>0.0122</u> | 79.2632 | <u>82.2496</u> |

Table 2: Performance comparison of image enhancement techniques on Brain MRI image using square pixel structure and hexagonal pixel structure

| Brain MRI Image Results | Mean Square Error (MSE) | | Bit Error | Rate (BER) | Peak Signal to Noise | | |
|-------------------------|-------------------------|-------------|-----------|-------------|----------------------|-----------|--|
| | | | | | Ratio (I | PSNR) | |
| | Square | hexagonal | Square | hexagonal | Square | hexagonal | |
| | pixel image | pixel image | pixel | pixel image | pixel image | pixel | |
| | | | image | | | image | |
| Single scale retinex | 0.0561 | 0.0558 | 0.0165 | 0.0165 | 60.6387 | 60.6634 | |
| Multiscale retinex | 0.0468 | 0.0463 | 0.0163 | 0.0163 | 61.4291 | 61.4712 | |
| Multiscale retinex with | 0.0042 | 0.0040 | 0.0139 | 0.0139 | 71.9246 | 72.0788 | |
| color restoration | | | | | | | |
| Homomorphic filtering | 0.0742 | 0.0852 | 0.0168 | 0.0170 | 59.4286 | 58.8259 | |
| Gabor filter | 0.0014 | 0.0013 | 0.0131 | 0.0130 | 76.5631 | 77.0756 | |

| Flower Image Results | Mean Square | Error (MSE) | Bit Error Rate (BER) | | Peak Signal to Noise Ratio (PSNR) | | |
|---|-------------|---------------|----------------------|---------------|--------------------------------------|----------------|--|
| | Square | hexagonal | Square | hexagonal | Square | hexagonal | |
| | pixer image | pixer image | image | image | pixer image | image | |
| Single scale retinex | 0.0196 | 0.0192 | 0.0153 | 0.0153 | 65.2152 | 65.3080 | |
| Multiscale retinex | 0.0152 | 0.0145 | 0.0151 | 0.0150 | 66.3192 | 66.5106 | |
| Multiscale retinex with color restoration | 0.0027 | 0.0029 | 0.0135 | 0.0136 | 73.8436 | 73.5126 | |
| Homomorphic filtering | 0.0851 | 0.0752 | 0.0170 | 0.0168 | 58.8326 | 59.3682 | |
| Gabor filter | 0.0014 | <u>0.0010</u> | 0.0130 | <u>0.0128</u> | 76.7770 | <u>78.0402</u> | |

Table 3: Performance comparison of image enhancement techniques on Flower image using square pixel structure and hexagonal pixel structure

| Table 4: Performance compar | ison of image | enhancement | techniques | on Face | image u | sing s | square |
|-------------------------------|-----------------|-------------|------------|---------|---------|--------|--------|
| pixel structure and hexagonal | oixel structure | | | | | | |

| Face Image Results | Mean squa | are error (MSE) Bit e | | rate (BER) | Peak signal to noise ratio (PSNR) | |
|------------------------|-----------|-------------------------------|--------|-------------|--------------------------------------|-------------|
| | Square | hexagonal | Square | hexagonal | Square | hexagonal |
| | pixel | pixel image | pixel | pixel image | pixel | pixel image |
| | image | | image | | image | |
| Single scale retinex | 0.0132 | 0.0152 | 0.0149 | 0.0151 | 66.9233 | 66.3164 |
| Multiscale retinex | 0.0039 | 0.0047 | 0.0139 | 0.0140 | 72.1866 | 71.3942 |
| Multiscale retinex | 0.0054 | 0.0058 | 0.0141 | 0.0142 | 70.7922 | 70.4906 |
| with color restoration | | | | | | |
| Homomorphic | 0.1570 | 0.1436 | 0.0178 | 0.0177 | 56.1717 | 56.5599 |
| filtering | | | | | | |
| Gabor filter | 0.0014 | <u>6.536 ×10⁻⁴</u> | 0.0130 | 0.0125 | 76.7593 | 79.9771 |

EXPERIMENTAL RESULTS & DISCUSSIONS

The performance of above mentioned image enhancement techniques has been evaluated on square and hexagonal pixels using four images of different types like brain MRI, foggy road, flower and face. Firstly, these square pixel images are converted to hexagonal pixel images using spiral addressing technique. Next step is image enhancement using single scale retinex, multi scale retinex, multi scale retinex with color restoration, homomorphic filtering, Gabor filter. Finally, the performance is compared in terms of mean square error (MSE), Bit error rate (BER) and peak signal to noise ratio (PSNR). Based on our results obtained after Enhancement of hexagonal pixel structure and square pixel structure; it has been proved that hexagonal pixel structure outperforms over square pixel structure. From each table it can be observed the maximum value of PSNR and minimum MSE, BER is obtained for Gabor Filter enhanced hexagonal pixel. Thus, it has been proved that Gabor filter enhancement of hexagonal pixels gives best results than square enhanced pixels in comparison with other enhancement techniques.

CONCLUSION

In this paper, we have briefly described the most famous enhancement techniques for digital images. A comparison of these techniques was performed on both square and hexagonal pixel images. The mathematical comparison was made for calculation of enhancement in term of PSNR, MSE and BER. This paper not only concludes with the enhancement results but also provides information that the hexagonal pixels provide good image quality than square pixels. Also this paper shows that the Gabor filter not only preserves the original Hue of the image but also gives maximum enhancement of the images compared with other enhancement techniques.

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